

UNIVERSITY OF CALIFORNIA

Los Angeles

An Evaluation of the Performance of the
Balanced Half-Sample and Jackknife Variance Estimation Techniques

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Biostatistics

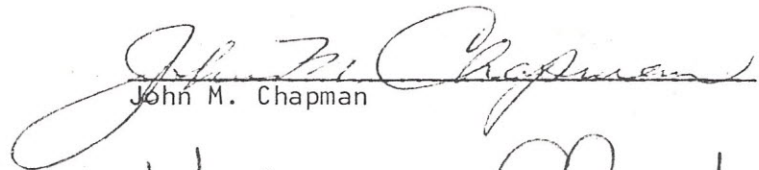
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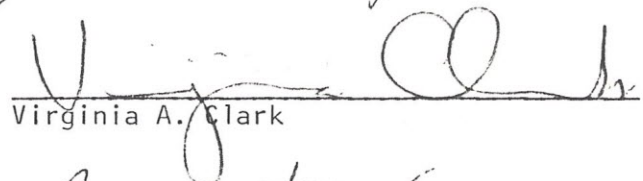
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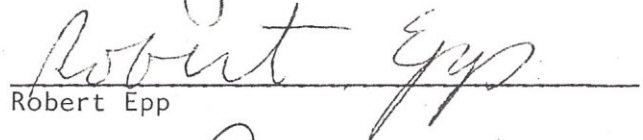
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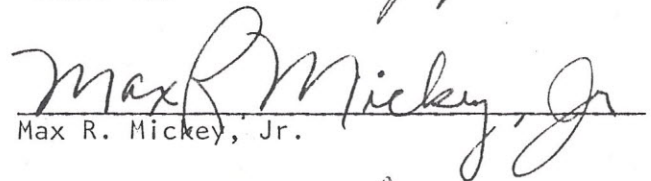
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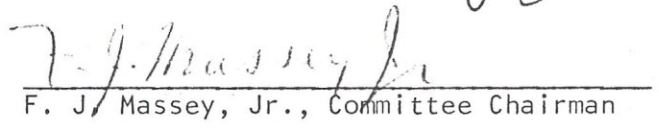
The dissertation of Stanley Lemeshow is approved, and it is acceptable in quality for publication on microfilm.


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Skinfold thickness in a national probability sample of U.S. males and
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ABSTRACT OF THE DISSERTATION

An Evaluation of the Performance of the
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by

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In recent years, methods for approximating the variances of estimates computed from complex sample surveys have received increased attention since the precise expressions for such variances are usually unknown. The balanced half-sample and jackknife are two such methods.

Considered in this dissertation are two balanced half-sample estimates ($\hat{V}_{B1}(\hat{W}), \hat{V}_{B2}(\hat{W})$) and three jackknife estimates ($\hat{V}_{J1}(\hat{W}), \hat{V}_{J2}(\hat{W}), \hat{V}_{J3}(\hat{W})$) of the variance of \hat{W} , where \hat{W} is the estimate of some parameter of interest. The particular situation considered is one in which the population is subdivided into L strata of known sizes or weights from which n_i observations are selected. Cases considered are such that, for each stratum, the n_i observations may be collapsed into k equal sized groups. For the balanced half-sample technique, $k=2$ groups of $r_i = n_i/2$ observations are always established, whereas for the jackknife method $2 \leq k \leq n_i$, each group containing $r_i = n_i/k$ observations.

The variance estimation techniques are introduced and theorems

are presented for the case where \hat{W} is a linear combination of the observations. In this linear situation the three jackknife estimators are identical for all values of k and L and are unbiased estimates of the true variance of \hat{W} . When L is a multiple of 4 and $k=2$, the value of $\hat{V}_{B2}(\hat{W})$ is identical to the three jackknife values but $\hat{V}_{B1}(\hat{W})$ differs from the rest and is negatively biased as an estimate of the true variance. However, its variance and mean square error is less than the corresponding values of any of the other estimates. When L is not a multiple of 4, all five estimates are identical when $k=2$. The variances and mean square errors of the jackknife estimates decrease as k increases. With $k > 2$, the variances and mean square errors of the jackknife estimates are less than the corresponding values for either of the balanced half-sample estimates.

As examples of non-linear \hat{W} , we consider the combined ratio estimate, the estimate of the slope in a linear regression situation, and the estimate of the correlation coefficient. The five variance estimators are evaluated by means of sampling experiments using computer generated data from populations with specified values for the strata parameters. The sampling experiment also produces estimates of the target variances so that bias and mean square error can be assessed.

The sampling experiments for the combined ratio estimate, \hat{R} , indicate that either the balanced half-sample or jackknife methods may be effectively used for the estimation of variances. There was close correspondence to theorems which were derived for the linear case. $\hat{V}_{B1}(\hat{R})$ was shown, on the average, to underestimate the target

variance when L was a multiple of 4. However, this estimate had lower variance and mean square error than the other four estimates when $k=2$. With $k>2$, the variance and mean square error of the jackknife variance estimates were smaller than the corresponding values of either balanced half-sample estimate.

Sampling experiments for the slope, β , and correlation coefficient, ρ , demonstrate that it is necessary to select a moderately large number of observations from each stratum in order to produce acceptable variance estimates. The jackknife estimates for $k>2$ are again shown to have smaller variance and mean square error than the corresponding balanced half-sample estimates. As was the case for the linear estimate or combined ratio estimate, $\hat{V}_{B1}(\hat{\beta})$ and $\hat{V}_{B1}(\hat{\rho})$ are, on the average, underestimates of the target variance but have smaller variance and mean square error than the other estimates when $k=2$ and L is a multiple of 4. However, contrary to the linear and combined ratio situations, there does not exist the high degree of similarity among the other variance estimation techniques. Using any of these methods, satisfactory estimates were made of the variance of the estimated slope or correlation coefficient provided that a large enough sample was selected from each stratum. Because of the flexibility afforded by the jackknife method to establish $k>2$ groups per stratum, a jackknife method would be preferred over a balanced half-sample method.

Also considered was the use of the variance estimation procedures when each sample individual is assigned a unique statistical weight. These weights presumably bring the demographic breakdown in the sample into closer alignment with known demographic characteristics of the

population. Two weighting schemes are considered for the purpose of estimating the variance of the estimated population mean. One method uses the same weights for estimates based on subgroups of the sample individuals (a half-sample is an example of such a subgroup) as are used for the estimate based on the entire sample. The other method assigns weights to the individuals in the subgroup so that the resulting estimate better reflects the population parameter. Examples are given of situations in which variance estimates produced using the latter method had markedly smaller bias and variance.